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Integrated Sachs-Wolfe effect in Cross-Correlation: The Observer's Manual

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The Integrated Sachs-Wolfe (ISW) effect is a direct signature of the presence of dark energy in the universe, in the absence of spatial curvature. A powerful method for observing the ISW effect is through cross-correlation of the Cosmic Microwave Background (CMB) with a tracer of the matter in the low redshift universe. In this letter, we describe the dependence of the obtained cross-correlation signal on the geometry and other properties of a survey of the low redshift universe. We show that an all-sky survey with about 10 million galaxies within $0 < z < 1$ should yield a near optimal ISW detection, at $\sim 5\sigma$ level. Then, we argue that, while an ISW detection will not be a good way of constraining the conventional properties of dark energy, it could be a valuable means of testing alternative theories of gravity on large physical scales.

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I. INTRODUCTION

One of the fundamental pieces of our understanding of modern cosmology comes from the study of anisotropies in the Cosmic Microwave Background (CMB). First discovered by the DMR experiment on the COBE satellite in the early 90's [1], the observations of CMB anisotropies matured through various ground-based/balloon-borne experiments (see [2] for a list) until its latest climax by the first year data release of the observations of the WMAP satellite [2] in 2003, a cosmic variance limited map of the CMB sky with a resolution of $\lesssim 0.5$ degs.

One of the less anticipated applications of the all-sky CMB map obtained by WMAP, is the study of a possible correlation between the CMB sky and the large scale structure (as measured, e.g. by galaxy the distribution) in the low-redshift universe. Such correlation could be due to secondary anisotropies [3] that are imprinted on the CMB sky, as the photons travel through the low-redshift universe. Following the WMAP data release, various groups [4] claimed a possible observation of such correlation with various galaxy surveys, although at a small $(2-3\sigma)$ significance level. Most recently, [5] claimed a correlation between WMAP maps and the 2MASS galaxy catalog, at both large and small angles.

In this letter, we focus on such correlations on angles larger than a few degrees, and its only known cosmological source, the Integrated Sachs-Wolfe (ISW) effect [6]. The possibility was first suggested by [7], and further explored theoretically in [8]. We first briefly review the physics of the expected signal and error in any ISW detection in cross-correlation. Then we consider over what redshift space, and what angular scales, most of the ISW signal arises, and how many galaxies a survey should have to overcome the Poisson noise in such a detection. Finally we discuss on what we may (or may not) learn from a detection of the ISW signal in cross-correlation. Through this letter, unless mentioned otherwise, we use the flat WMAP+CBI+ACBAR+2dF+Ly-α concordance cosmological model [9], a cosmological constant (i.e. $w = -1$), and a running spectral index.

II. THE ISW EFFECT IN CROSS-CORRELATION

The ISW effect is caused by the time variation in the cosmic gravitational potential, Φ, as CMB photons pass through it. In a flat universe, the anisotropy in CMB photon temperature due to the ISW effect is an integral over the conformal time η

$$\delta_{\rm ISW}(\hat{\mathbf{n}}) = \frac{\delta T_{\rm ISW}}{T} = 2\int \Phi^{'}[(\eta_0-\eta)\hat{\mathbf{n}},\eta]\; d\eta, \qquad (1)$$

where $\Phi^{'} \equiv \partial\Phi/\partial\eta$, and $\hat{\mathbf{n}}$ is the unit vector along the line of sight. The linear metric is assumed to be

$$ds^2 = a^2(\eta)\{[1+2\Phi(\mathbf{x},\eta)]d\eta^2 - [1-2\Phi(\mathbf{x},\eta)]\mathbf{dx}\cdot\mathbf{dx}\}, \qquad (2)$$

(the so-called longitudinal gauge) and η_0 is the conformal time at the present.

In a flat universe, Φ does not change with time, at any given comoving point, for a fixed equation of state and therefore observation of an ISW effect is an indicator of a change in the equation of state of the universe. Assuming that this change is due to an extra component in the matter content of the universe, the so-called dark energy, this component should have a negative pressure to become important at late times [10]. Therefore, observation of an ISW effect in a flat universe is a signature of dark energy.

The ISW effect is observed at large angular scales because most of the power in the fluctuations of Φ is at

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large scales. Additionally, the fluctuations at small angles tend to cancel out due to the integration over the line of sight.

We are interested in finding the cross-correlation of the ISW effect with the galaxy distribution. Assuming Gaussian initial conditions, and full-sky coverage, different harmonic multipoles are statistically independent in the linear regime. Therefore, as the ISW effect is only important on large scales which are still linear, the statistical analysis is significantly simplified in harmonic space. For a galaxy survey with the average comoving density distribution $n_c(r)$ as a function of the comoving distance r, the Limber equation can be used to approximately describe the expected cross-correlation with the galaxy distribution [20] (see [5] for detailed derivation)

$$C_{gT}(\ell) \equiv \langle \delta^{\rm 2D}_{g,\ell m} T_{\ell m} \rangle = \frac{2}{\int dr\ r^2 n_c(r)} \int dr\ n_c(r) P_{\Phi',g}\left(\frac{\ell+1/2}{r}\right), \quad (3)$$

where $\delta^{\rm 2D}_{g,\ell m}$ and $T_{\ell m}$ are the projected survey galaxy overdensity and the CMB temperature in the spherical harmonic space, while $P_{\Phi',g}(k)$ is the 3D cross-power spectrum of Φ' and galaxy overdensity, for wave-number k. Assuming that the galaxies follow matter density with a constant bias b_g, i.e. $\delta_g = b_g \delta_m$, we can solve the G_{00} Einstein equation [11]

$$(k^2+3\mathcal{H}^2)\Phi + 3\mathcal{H}\Phi' + 4\pi G a^2(\rho_m\delta_m + \rho_{DE}\delta_{DE}) = 0, \quad (4)$$

to relate $P_{\Phi',g}$ to the matter auto-power spectrum $P_{m,m}(k) = P(k) = |\delta^2_m(k)|$, where $\mathcal{H} = d\ln a/d\eta$ is the conformal Hubble constant, and ρ_{DE} and δ_{DE} are the average density and overdensity of the dark energy, respectively. Note that, for a cosmological constant (a ΛCDM cosmology), $\delta_{DE} = 0$. For any alternative theory of dark energy, an independent equation for the evolution of δ_{DE} should be solved simultaneously.

III. THE ERROR IN ISW DETECTION

It is easy to see that the expected dispersion (see [5] for details [21]) in the cross-correlation signal for harmonic multipole $C_{gT}(\ell)$ is given by

$$\Delta C^2_{gT}(\ell) \simeq \frac{C_{gg}(\ell) C_{TT}(\ell)}{f_{\rm sky}(2\ell+1)}, \quad (5)$$

where $f_{\rm sky}$ is the fraction of sky covered in the survey, and we assumed a small cross-correlation signal, i.e. $C^2_{gT}(\ell) \ll C_{gg}(\ell)C_{TT}(\ell)$, which is the case for the ISW effect (the ISW effect is much smaller than the primary anisotropies, but see[12]).

$C_{TT}(\ell)$ is the observed CMB temperature auto-power, which includes both the intrinsic CMB fluctuations and the detector noise. As becomes clear later on, since the ISW effect is observed at small ℓ, the WMAP observed auto-power spectrum[13], which has negligible detector noise at low ℓ, should give the optimum power spectrum which can be used in Eq.(5). We again use the Limber approximation to obtain the projected galaxy auto-power

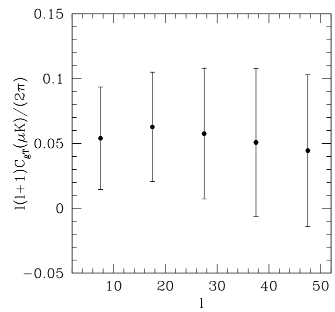

FIG. 1: The expected cross-power spectrum of the ISW effect, for an all-sky survey with $b_g^2 dN/dz = 10^7$, and $z_{\rm max} = 1$

$$C_{gg}(\ell) \simeq \frac{\int dr\ r^2\ n_c^2(r)[b_g^2(r)\cdot P\left(\frac{\ell+1/2}{r}\right) + n_c^{-1}(r)]}{\left[\int dr\ r^2\ n_c(r)\right]^2}. \quad (6)$$

Fig. (1) shows an example of the expected cross-power signal and error for a survey with 10 million galaxies with $b_g = 1$, between $0 < z < 1$.

IV. THE PERFECT GALAXY SURVEY

To obtain the optimum signal-to-noise ratio for an ISW detection in cross-correlation, we assume that we have an (at least approximate) redshift estimate for each galaxy in the survey. Then we can divide the survey into almost independent shells of thickness δr, where

$$r_0 \ll \delta r \ll r,$$

where $r_0 \lesssim 5h^{-1}\,{\rm Mpc}$, is the galaxy auto-correlation length. Combining equations (3,5,6) for a thin shell, we find the expected signal-to-noise ratio for the cross-correlation signal in multipole ℓ, due to this shell

$$\delta(S/N)^2 = \frac{C^2_{gT}(\ell)}{\Delta C^2_{gT}(\ell)} = \frac{\left[f_{\rm sky}\cdot r^2\delta r\cdot(2\ell+1)\right]\times 4P^2_{\Phi',m}(k)}{C_{TT}(\ell)[P(k)+(n_c b_g^2)^{-1}]}, \quad (7)$$

where $k = \frac{\ell+1/2}{r}$. Within the approximation of independent shells and multipoles, $(S/N)^2$ is cumulative, and we could simply add (or integrate over) the contribution due to different multipoles and shells that are included in the galaxy survey, and multiply it by the sky coverage, $f_{\rm sky}$, to obtain the optimum $(S/N)^2$ (in the absence of systematics) for the whole survey. Fig.(2) shows the $(S/N)^2$ density distribution for hypothetical all-sky surveys with limited (CMB) resolution, or redshift depth, in a ΛCDM cosmology, while we assumed that the Poisson noise is negligible ($n_c \to \infty$).

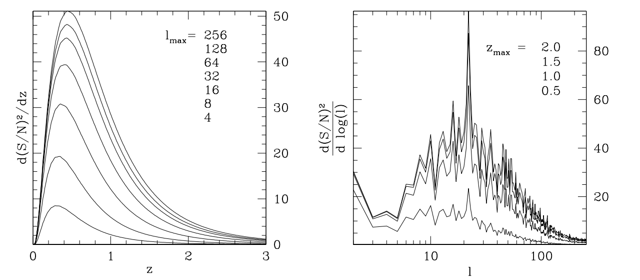

FIG. 2: Figures show the expected $(S/N)^2$ distribution for a redshift or resolution limited full-sky ISW cross-correlation signal ($\ell_{\max}$ refers to the scale at which detector noise is equal to the true signal). In either figures, the enclosed area for the region covered by a survey, multiplied by its sky coverage, gives the optimum $(S/N)^2$ for the cross-correlation signal. The spikiness of the distribution in ℓ-space is due to the use of actual observed WMAP power spectrum in Eq.(7). Note that for a partial sky coverage, the low-ℓ multipoles that are not covered by the survey should also be excluded from the area.

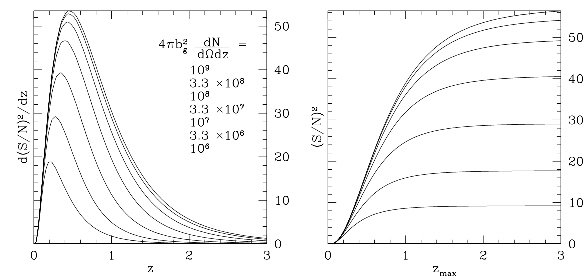

FIG. 3: The distribution of $(S/N)^2$ for different galaxy numbers per unit redshift (left). The total $(S/N)^2$ for a fixed dN/dz up to $z_{\max}$ (right). For partial sky coverage, the result should be multiplied by $f_{\rm sky}$.

We see that the ISW cross-correlation signal is widely distributed over a redshift range between 0 and 1.5 (which is the era in which dark energy becomes cosmologically important), and peaks at $z \simeq 0.4$, below which the detection is limited by the available volume. Almost all the signal is due to multipoles with $\ell \lesssim 100$ ($\theta \gtrsim 2°$), which implies that the WMAP 1st year all-sky temperature map [2], with a resolution of less than a degree ($\ell > 200$), captures all the ISW signal in the CMB sky. The angular scale of the cross-correlation signal decreases with the depth of the survey. This is due to the fact that the angular correlation length of the galaxy distribution is smaller for a deeper (more distant) sample.

For our assumed ΛCDM cosmological model, the total S/N which could be achieved by a perfect survey is ~ 7.5. Since the ISW kernel has such a broad distribution in the redshift space, even large errors in the estimated redshifts ($\Delta z \sim 0.5$) will not decrease the signal-to-noise by more than 5%. Therefore, photometric redshift estimates will be adequate for any ISW cross-correlation work.

V. POISSON LIMITED SURVEYS

For a realistic survey, an additional source of noise are Poisson fluctuations in the galaxy number density. The Poisson noise (the second term in the brackets in Eq. 6) is inversely proportional to the average number density of observed galaxies in the survey, and thus, dominates the uncertainty in cross-correlation for a small galaxy sample. Fig. (3) shows how the total cross-correlation $(S/N)^2$ and its distribution depend on the number of galaxies in an all-sky survey. Although a fixed number of galaxies per unit redshift is assumed, different curves in the Fig. (3;left) could be combined to obtain the signal-to-noise for an arbitrary redshift/bias distribution.

Fig.(3) shows that an ambitious all-sky survey with about 10 million galaxies (or one million clusters with $b_g \sim 3$), which cover the redshift range between 0 to 1, can only yield a S/N of $\simeq 5$. Although Sloan Digital Sky Survey (SDSS) is unlikely to get a S/N better than 4σ due to its incomplete redshift coverage, future all-sky galaxy surveys like LSST and Pan-STARRS are bound to achieve an almost perfect detection of ISW in cross-correlation[14], should they cover the whole sky.

VI. WHAT DOES ISW TELL US ABOUT COSMOLOGY?

Let us study the optimum constraints that an ISW cross-correlation detection can give us about cosmology. For a given cosmological model $\mathcal{C}$, using Eq. (3), the expected ISW cross-power signal for a narrow redshift bin $(z, z+\delta z)$, at multipole ℓ, is

$$C_{gT}(\ell, z; \mathcal{C}) = \frac{2b_g(r)}{r^2(z)} P_{\Phi',m}\left(\frac{\ell+1/2}{r}; \mathcal{C}\right), \qquad (8)$$

while combining Eqs.(5,6) gives the error in the cross-power for a perfect survey

$$\Delta C^2_{gT}(\ell, z; \mathcal{C}) = \frac{C_{TT}(\ell) b_g^2(r)}{(2\ell+1) r^2 \delta r} P\left(\frac{\ell+1/2}{r}; \mathcal{C}\right). \qquad (9)$$

Then, for a nominal concordance cosmology $\mathcal{C}_0$, the expected significance level for ruling out the cosmology $\mathcal{C}$, χ, is given by

$$\chi^2 = \sum_{\ell,z} \frac{[C_{gT}(\ell, z; \mathcal{C}) - C_{gT}(\ell, z; \mathcal{C}_0)]^2}{\Delta C^2_{gT}(\ell, z; \mathcal{C}_0)}. \qquad (10)$$

Note that the bias factor, b_g, is cancelled from the numerator and the denominator, and the optimum significance level only depends on the fluctuations in matter density and gravitational potential.

Fig. (4) shows the optimum constraints that an ISW detection may yield on some of the properties of dark

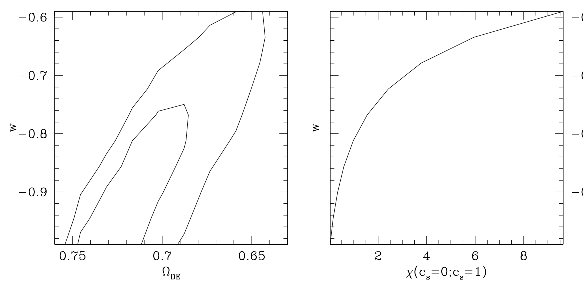

FIG. 4: (Left) The $w - \Omega_{DE}$ constraints (68% and 95% contours) based on an optimum ISW detection. (Right)The significance of ruling out $c_s = 0$, for a quintessence model with given w, and $c_s = 1$, based on an optimum ISW detection. In both graphs, the other cosmological parameters are kept constant.

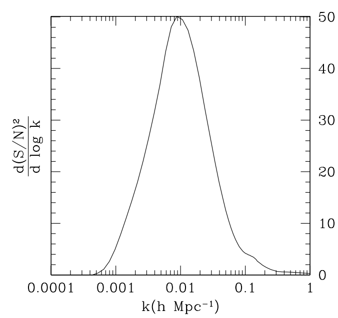

FIG. 5: The distribution of $(S/N)^2$ of an ISW detection in k-space.

energy, i.e. its density, equation of state, and speed of sound[16]. While such constraints are already comparable to the current bounds on these parameters [9, 15, 17], future observations of CMB and large scale structure [18] will significantly improve these bounds. However, ISW constraints could still be used to test possible systematics that may affect the accuracy of future measurements.

A more intriguing application of an ISW detection is the possibility of testing our theories of matter/gravity on large scales. While the consistency of the power spectrum of 2dF and SDSS galaxy surveys [19] with the WMAP power spectrum confirms the consistency of the ΛCDM concordance cosmology at scales of $k \gtrsim 0.1\ h\ \mathrm{Mpc}^{-1}$, Fig.(5) shows that an ISW detection can do the same at scales of $k \sim 0.003 - 0.03\, h\, \mathrm{Mpc}^{-1}$. Therefore, current [4, 5] and future observations of an ISW effect may confirm the consistency of our cosmological theories at the largest physical scales that they have ever been tested. While the cross-correlation statistics is almost free of systematic bias, the auto-correlation is often dominated by survey systematics on such scales.

VII. DISCUSSION

In this letter, we study different aspects of the correlation between a galaxy survey and CMB sky, due to the ISW effect. The main source of noise is contamination by the primary CMB anisotropies. We see that, given this noise, most of the signal comes from $\ell \sim 20$, and $z \sim 0.4$ with negligible contribution from $\ell > 100$ and $z > 1.5$. An all-sky survey with about 10 million galaxies within $0 < z < 1$ should yield an almost perfect ISW detection, at the $\sim 5\sigma$ level.

It turns out that, due to the large noise induced by the primary anisotropies, the optimum constraints on the properties of dark energy from an ISW detection, are already comparable to the current accuracies, and will be outdone by future observations. However, the simplicity of the linear physics involved in the ISW effect, and the fact that the cross-correlation statistics are not biased by the systematics of CMB or galaxy surveys, makes ISW detection a useful indicator of possible systematics in more accurate methods.

Finally, we point out that the detection of the ISW effect provides a unique test of our concordance cosmological model on the largest physical scales that it has ever been tested.

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[21] If the survey does not cover the whole sky, the nearby multipoles in the harmonic space will not be independent. However, if we average over a large number of multipoles, we could assume equation (5) for the standard deviation of each multipole and treat them as independent.